ACCEPTANCE OF SECONDARY PARTICLE BEAMS

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Of the total yield of charged particles within a given momentum band, produced in a target, some fraction will enter the defining aperture of any given secondary beam transport system. The parameters that determine how large this fraction will be are the angular distribution of the secondaries, the mean production angle of the accepted particles, and the size of the aperture.

The angular distribution of secondary particles is not very accurately known, especially at small angles or high momenta. In the absence of reliable data one usually chooses as guide one of several empirical or phenomenological distributions, which fit the available data more or less passably. The purpose of the present note is to present in particularly convenient form the predictions of these distributions, and to allow in addition the use of any other arbitrary distribution. The results are given in the form of curves, showing the fraction of all particles produced (in a given momentum interval) that will be accepted in an aperture of given size, at a given mean production angle. The major uses are to evaluate the importance of small production angles, by showing how the yield falls off with angle, and also the beam intensity to be expected.

I. Normalization of the Angular Distribution.

To carry out the proposed program it is convenient to normalize the entire emitted flux of particles of a given momentum to unity. For the case of an unpolarized beam striking an unpolarized target (so that the distribution has no dependence on azimuthal angle), the yield function is a function of the polar angle x only. If the yield is Y(x), the flux normalization condition is

$$\int_0^\infty 2\pi x Y(x)dx = 1. \tag{1}$$

In addition it will be useful to normalize the angular variable x so that its mean value corresponds to the observed mean angle of particle emission. The experimental data indicate that to a good approximation, particles of all sorts produced at any energy have the same mean transverse momentum, i.e. about .35 GeV/c, and an approximately Boltzmannian distribution in angle. Consequently, if we normalize the angular distribution so that the mean value of x, i.e. $\bar{x}$, is unity, and let x=1 represent a transverse momentum of .35 GeV/c, a single distribution will represent all momenta, and a single set of curves can be used for all momenta.

The angular normalization requirement may then be written

$$\bar{x} = \frac{\int_0^\infty 2\pi x^2 Y(x)dx}{\int_0^\infty 2\pi x Y(x)dx} = 1 \tag{2}$$

The two normalization conditions (1) and (2) then serve to determine two arbitrary constants in any angular distribution formulas.

II. Analysis of Angular Distributions.

Four principal forms of angular distribution have been proposed to describe high energy particle production. These include the CKP[1], which is a Boltzmann-like distribution in transverse momentum; the Trilling[2], which is an attempt to improve on the CKP, and differs from it only in providing a proper cutoff at the high-energy end, and in its predictions for the lowest third of the momentum range. The Hagedorn-Ranft[3] distribution is based on a statistical fireball model, and is intended for p-p, not p-nucleus collisions. A gaussian form has also been suggested[4].

Since the Trilling distribution does not differ from the CKP over a large range, and the Hagedorn-Ranft is inapplicable, we evaluate the CKP and gaussian distributions. They have the functional dependences:

$$Y_{CKP}(x) = A \exp(-bx) \tag{3}$$

and

$$Y_{gauss}(x) = C \exp(-dx^2) \tag{4}$$

These expressions contain only two arbitrary constants each, and therefore the normalization procedures will determine them completely. Carrying out the normalization, we obtain

$$Y_{CKP}(x) = \frac{2}{\pi} \exp(-2x) \tag{5}$$

$$Y_{gauss}(x) = \frac{1}{4} \exp\left(-\frac{\pi x^2}{4}\right) \tag{6}$$

III. Calculation of the Acceptance of a Circular Aperture.

Given the normalized angular distribution, it is not difficult to calculate the fraction of the beam that will enter an aperture of given shape and size, as a function of angular position. For any given momentum, the angle is determined by the condition that x=1 represents a transverse momentum of .35 GeV/c. The radius of a circular aperture may then be specified in the same units as x, and its center located, in the same units.

To solve this problem for circular apertures, a computer program in BASIC, called AP591, has been written. The program accepts any arbitrary angular distribution. The results of computations for the normalized distributions given in (5) and (6) are given in Figs. 1-3. A later version of the program that treats of elliptical apertures is in preparation.

REFERENCES

[1] Cocconi, Koester, and Perkins, Lawrence Radiation Laboratory Report UCRL-10022, 167(1961): G. Cocconi, Proc. Argonne Users Group Meeting, Dec. 1961.

[2] G. Trilling, UCID-10418, Lawrence Radiation Laboratory Report UCRL-16830, 200-BeV Summer Study Vol. I, 1964-65, p. 25.

[3] R. Hagedorn and J. Ranft, CERN TH/851, Dec. 1967.

[4] Ratner, Edwards, Akerlof, Crabb, Day, Krisch and Lin, Phys. Rev. Letters 18, 1218 (1967).

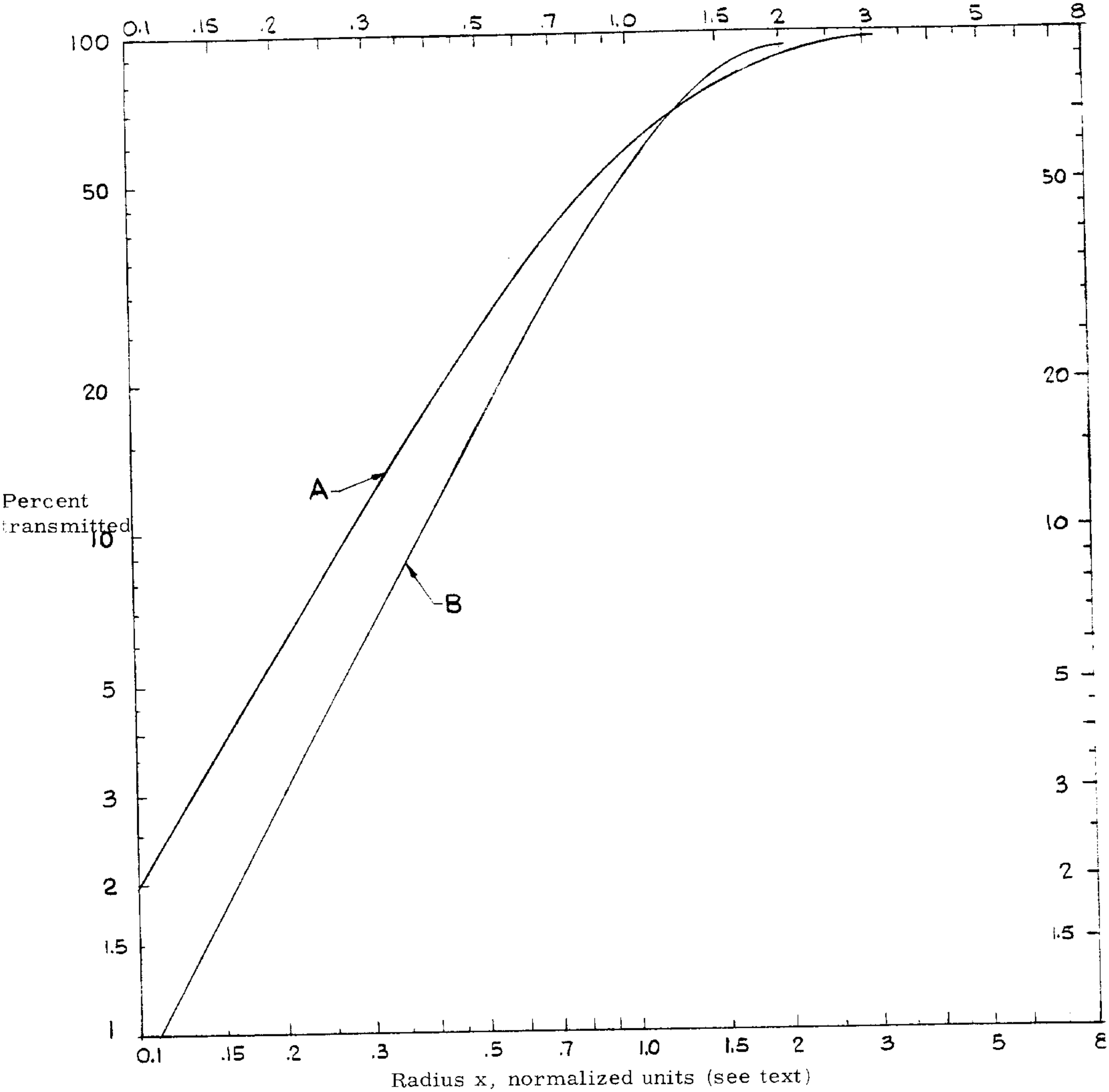
FIGURE CAPTIONS

Fig. 1(a). Fraction of the total beam transmitted through a circular aperture of radius x for the normalized CKP distribution, for zero production angle.

(b). The same, for the normalized gaussian distribution.

Fig. 2. Transmission through three different circular apertures, for the CKP distribution as a function of transverse momentum, in normalized units; x = 1 represents about .35 GeV/c transverse momentum. The aperture radii for the cases a, b, and c are R = 0.8, 0.4, and 0.2 respectively, in the same units of angle as the transverse momentum. (E. g., R = 0.2 means an aperture whose angular radius is one fifth the angle at which the transverse momentum is .35 GeV/c).

Fig. 3. The same as Fig. 2, for radii 1.25, .8, .4, and .2 respectively, and gaussian angular distribution.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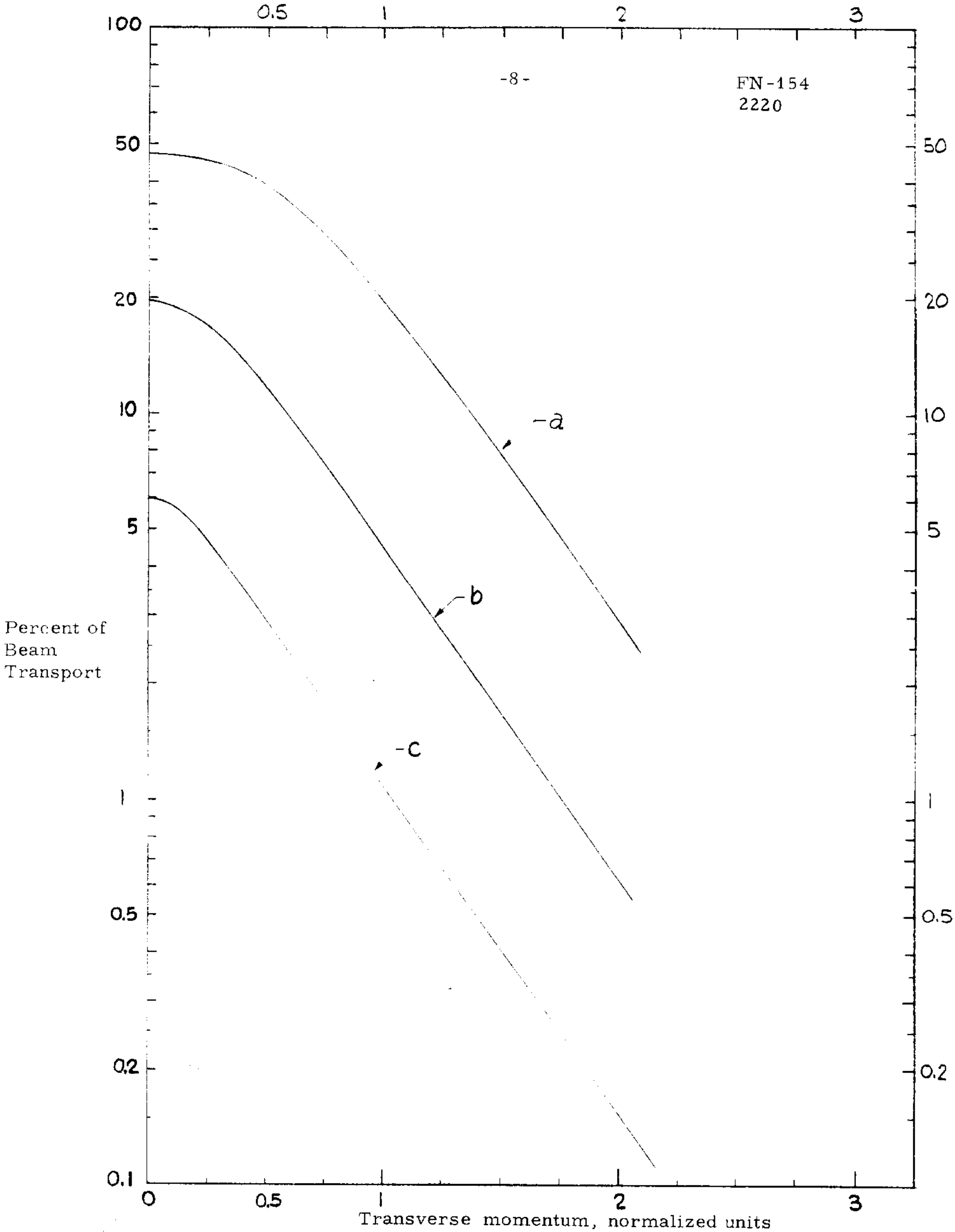
A
B
Percent transmitted
Radius x, normalized units (see text)

Fig. 1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-a
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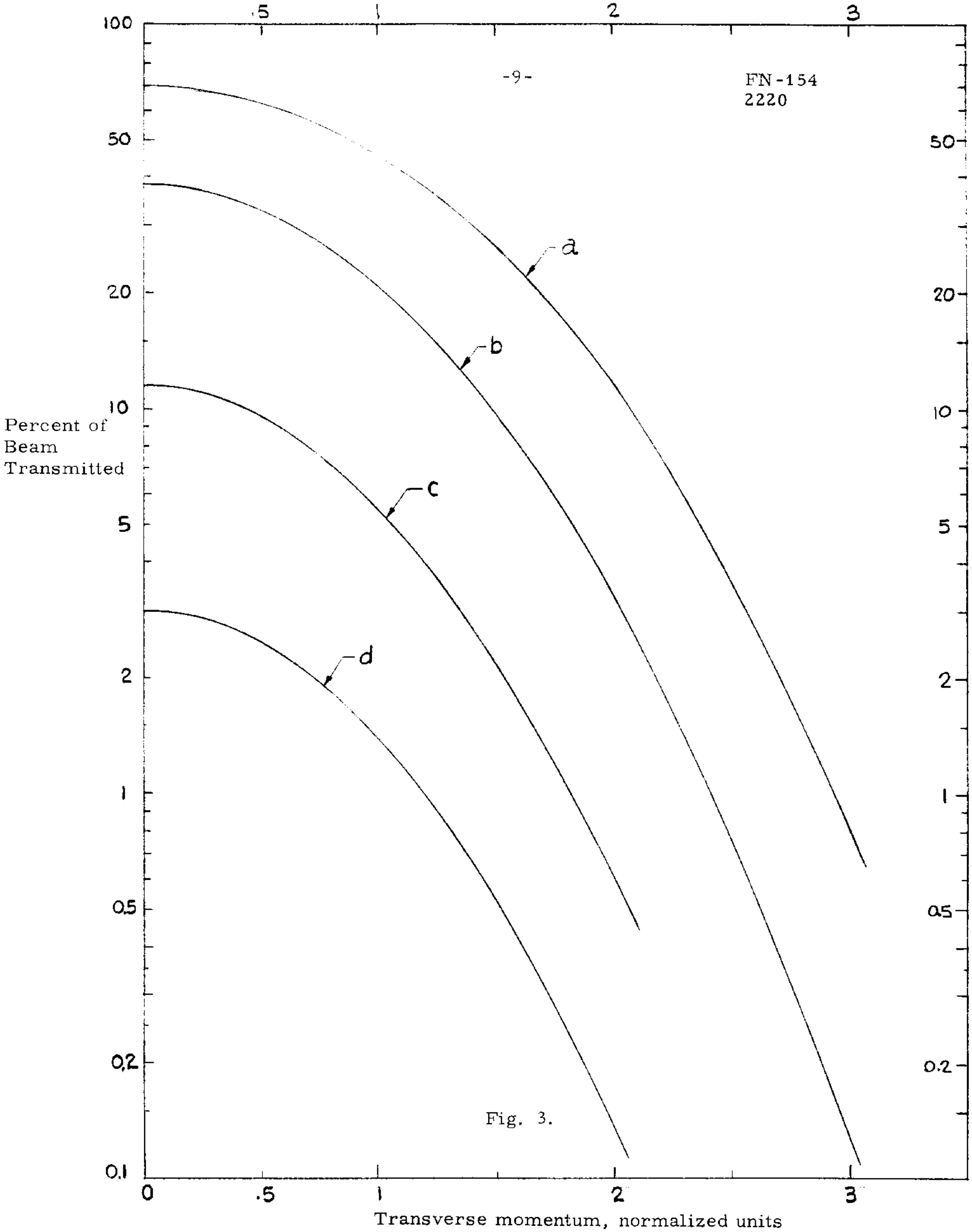
-c
Percent of
Beam
Transport
0
0.5
1
2
3
Transverse momentum, normalized units

Fig. 2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0.2
0.1
a
b
c
d
Percent of Beam Transmitted
0
Transverse momentum, normalized units

Fig. 3.